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## SOLUTION BY THE PROPOSER.

Let  $\sin A = a$ , or  $A = \sin^{-1} a$ , and let  $b = \sin(A + nh) = \sin B$ . Hence  $n = (B - A)/h$ .

Then we have

$$\begin{aligned}
 & \frac{a}{\sin A \sin(A+h) \cdots \sin[A+(n-1)h] \sin(A+nh)} \frac{b}{\sin(A+nh)} \\
 \int_a^b \sin^{-1} x dx &= \lim_{h \rightarrow 0} [(\sin(A+h) - \sin A) \sin^{-1} \sin A \\
 &\quad + (\sin(A+2h) - \sin(A+h)) \sin^{-1} \sin(A+h) + \cdots \\
 &\quad + (\sin(A+nh) - \sin[A+(n-1)h]) \sin^{-1} \sin[A+(n-1)h]] \\
 &= \lim_{h \rightarrow 0} [-h \{\sin(A+h) + \sin(A+2h) + \cdots + \sin(A+nh)\} - A \sin A \\
 &\quad + (A+nh) \sin(A+nh)] \\
 &= \lim_{h \rightarrow 0} \left[ -\frac{h/2}{\sin h/2} \left\{ \cos\left(A+\frac{h}{2}\right) - \cos\left[A+(2n+1)\frac{h}{2}\right] \right\} - A \sin A \right. \\
 &\quad \left. + (A+nh) \sin(A+nh) \right] \\
 &= \lim_{h \rightarrow 0} \left[ -\frac{h/2}{\sin h/2} \left\{ \cos\left(A+\frac{h}{2}\right) - \cos\left(B+\frac{h}{2}\right) \right\} - A \sin A + B \sin B \right] \\
 &= \cos B - \cos A + B \sin B - A \sin A \\
 &= \sqrt{1-b^2} - \sqrt{1-a^2} + b \sin^{-1} b - a \sin^{-1} a,
 \end{aligned}$$

which agrees with the result given in the tables.

An excellent solution was also received from A. M. HARDING.

## 367. Proposed by C. N. SCHMALL, New York City.

Show that the volume inclosed by the surface  $(x^2 + y^2 + z^2)^5 = (a^3 x^2 + b^3 y^2 + c^3 z^2)^2$  is  $\frac{4}{3}\pi(a^3 + b^3 + c^3)$ .

SOLUTION BY A. M. HARDING, University of Arkansas, AND A. R. NAUER, St. Louis, Mo.

Let  $x = r \sin \varphi \cos \theta$ ,  $y = r \sin \varphi \sin \theta$ ,  $z = r \cos \varphi$ . Substituting in the given equation, we obtain

$$r^3 = a^3 \sin^2 \varphi \cos^2 \theta + b^3 \sin^2 \varphi \sin^2 \theta + c^3 \cos^2 \varphi.$$

Then the volume inclosed by the surface is given by

$$\begin{aligned}
 V &= 4 \int_0^{\pi/2} \int_0^\pi \int_0^r r^2 \sin \varphi dr d\theta d\varphi, \\
 &= \frac{4}{3} \int_0^{\pi/2} \int_0^\pi (a^3 \sin^2 \varphi \cos^2 \theta + b^3 \sin^2 \varphi \sin^2 \theta + c^3 \cos^2 \varphi) \sin \varphi d\theta d\varphi, \\
 &= \frac{4}{3} \int_0^{\pi/2} \left( \frac{\pi}{2} \cdot a^3 \sin^3 \varphi + \frac{\pi}{2} b^3 \sin^3 \varphi + \pi c^3 \cos^2 \varphi \sin \varphi \right) d\varphi, \\
 &= \frac{4\pi}{3} \left[ \frac{a^3}{2} \cdot \frac{2}{3} + \frac{b^3}{2} \cdot \frac{2}{3} + \frac{c^3}{3} \right] = \frac{4\pi}{9} (a^3 + b^3 + c^3).
 \end{aligned}$$